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Classes of Mappings in Metric Spaces

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Abstract: The aim of this paper is to present certain basic properties of some class of mappings called (m,∞)-expansive and (m,∞)-contractive mappings acting on a real metric space.

Keywords: m-isometry; expansive map; contractive map; metric space

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1. Introduction and Preliminaries

The introduction of the concept of m-isometric transformation in Hilbert spaces by Agler and Stankus yielded a flow of papers generalizing this concept both in Hilbert and Banach spaces. For more details see [5, 6, 7, 8, 11, 16] and the references therein. In [19, 20] the second named author has introduced and studied several results on (m; p)-(hyper)expansive and (m; p)-(hyper)contractive maps on a metric space. These concepts extend the definitions of m-isometry on a Hilbert space and also on a Banach space. The results follow the trend of some recent research by T.Berm´udez and others.

As noted in [2] an operator acting on a Hilbert space $\mathscr{H}$ is called m-isometries for some integer $m \geq 1$ if

$$\sum_{0\leq k\leq m}(-1)^k\binom{m}{k}T^{*m-k}T^{m-k}=0 \tag{1.1}$$

or equivalently

$$\sum_{0\leq k\leq m}(-1)^k\binom{m}{k}\|T^{m-k}x\|^2=0 \qquad \forall\, x\in X. \tag{1.2}$$

The equation (1.2) was used to define m-isometries on a Banach space by Sid Ahmed [16] and by Botelho [7]. Bayart [5] has replaced the exponent 2 in (1.2) by an $p\in[1,\infty)$ and was introduced the following definition: a bounded linear operator $T: X\to X$, on a Banach spaces X is an (m,p)-isometry ($m\geq 1$ integer, $p\geq 1$ real) if

$$\sum_{0\leq k\leq m}(-1)^k\binom{m}{k}\|T^{m-k}x\|^p=0 \qquad \forall\, x\in X. \tag{1.3}$$

Hoffman et al. [12] considered the above definition with $p>0$ real and studied the role of the second parameter p and also discussed the case $p=\infty$.

Let $T,A\in\mathscr{B}(\mathscr{H})$ where A is positive. T is said to be (A,m)-isometry if T satisfies

$$\sum_{0\leq k\leq m}(-1)^{m-k}\binom{m}{k}T^{*k}AT^k=0,$$

for some positive integer m. (See [21, 22]).

Let X and Y be metric spaces. A mapping $T : X \longrightarrow Y$ is called an isometry if it satisfies

$$d_Y(Tx,Ty) = d_X(x,y), \forall x,y \in X,$$

where $d_X(.,.)$ and $d_Y(.,.)$ denote the metrics in the spaces X and Y, respectively.

In [6] it was introduced a notion of (m,p)-isometry for maps on a metric space: a map $T : X \to X$, on a metric space X with distance d, is called an (m,p)-isometry ($m \geq 1$, integer, $p > 0$ real) if it satisfies

$$\sum_{0\leq k\leq m} (-1)^k \binom{m}{k} d\left(T^{m-k}x, T^{m-k}y\right)^p = 0, \ \ \forall \ x,y \in X. \tag{1.4}$$

The Concept of *completely hyperexpansive* operators on Hilbert space has attracted much attention of various authors (see [4] and [23]). For this, it is important to study m-expansive operators. We refer the reader to [9] and [13] for more information about m-expansivity. Recently, the concept of (m,p)-expansive,(m,p)-contractive, (m,p)-hyperexpansive and (m,p)-hypercontractive operators on a Banach space has introduced and studied in slightly generalised form in [19, 20] and C.Gu [10]. We quote the definition given in [19, 20]: Fix a Banach space operator A on a Banach space X. For a bounded linear operator T acting on a Banach space X, we denote

$$\Theta_l^{(p)}(A,T,x) := \sum_{0\leq j\leq m} (-1)^j \binom{l}{j} \|AT^jx\|^p, \qquad \forall \ x \in X, \tag{1.5}$$

where $l \in \mathbb{N}_0$ is a integer, $p \in (0,\infty)$ and $\binom{l}{k}$ denotes the binomial coefficient. The operator T is said to be $A(m,p)$*-expansive* if $\Theta_m^{(p)}(A,T,x) \leq 0$. When such a relation is valid for $k \in \{1,2,...,m\}$, we say that T is $A(m,p)$*-hyperexpansive*. Moreover if $\Theta_m^{(p)}(A,T,x) \geq 0$, we say that T is $A(m,p)$-contractive and if T is $A(k,p)$-contractive for all positive integer $k \leq m$, the map T is said $A(m,p)$*-hypercontractive*. If $\Theta_m^{(p)}(A,T,x) = 0$ for all x, the operator T is said to be an $A(m,p)$*-isometry* (concept introduced and studied by B.P.Dugal in [8]).

Very recently, in paper [19], the second named author introduced and studied a class of mappings acting on a metric space,called (m,p)-expansive and (m,p)*-hyperexpansive*. Given an map $T : X \to X$ where (X,d) is a metric space, set

$$\Theta_l^{(p)}(d,T;x,y) := \sum_{0\leq k\leq l} (-1)^k \binom{l}{k} d\left(T^kx, T^ky\right)^p, \forall x,y \in X, \tag{1.6}$$

where $l \in \mathbb{N}_0$ is a integer, $p \in (0,\infty)$. The defining inequality of (m,p)*-expansive* $\big($resp.(m,p)*-hyperexpansive*$\big)$ mapping is $\Theta_m^{(p)}(d,T;x,y) \leq 0$ $\big($ resp.$\Theta_k^{(p)}(d,T;x,y) \leq 0$ for $k \in \{1,2,...,m\}\big)$.

Let $m \in \mathbb{N}$, an operator T acting on a Banach space X is called an (m,∞)-isometry (or (m,∞)-isometric operator) if and only if

$$\max_{\substack{k \in \{0,1,\cdots,m\} \\ k \text{ even}}} \|T^k x\| = \max_{\substack{k \in \{0,1,\cdots,m\} \\ k \text{ odd}}} \|T^k x\|, \quad \forall\, x \in X.$$

(See [12]).

Very recently, in paper [3], the author studied a classes of mappings acting on a metric space, called (m,∞)-isometries. An mapping T acting on a metric space (X,d_X) is called an (m,∞)-isometry for some positive integer m, if for all $x,y \in X$

$$\max_{\substack{0 \le k \le m \\ k \text{ even}}} d_X\left(T^k x, T^k y\right) = \max_{\substack{0 \le k \le m \\ k \text{ odd}}} d_X\left(T^k x, T^k y\right).$$

In the present paper, we present some basic properties of (m,∞)-expansive,(m,∞)-hyperexpensive,(m,∞)-contractive and (m,∞)-hypercontractive mappings on a metric spaces.

Throughout this paper, $\mathbb{R}$ denotes the field of real numbers. The natural numbers $\{1,2,3,\ldots\}$ are denoted by $\mathbb{N}$ and the non-negative integers by $\mathbb{N}_0$.

2. (m,∞)-expansive and (m,∞)-contractive mappings

In this section, we study (m,∞)-expansive and (m,∞)-contractive mappings.

A self mapping T on a metric space (X,d) is (m,p) expansive if and only if

2. (m,∞)-expansive and (m,∞)-contractive mappings

In this section, we study (m,∞)-expansive and (m,∞)-contractive mappings.

A self mapping T on a metric space (X,d) is (m,p) expansive if and only if

$$\sum_{0 \le k \le m} (-1)^k \binom{m}{k} d(T^k x, T^k y)^p \le 0$$

$$\Leftrightarrow \sum_{\substack{0 \le l \le m \\ k \text{ even}}} \binom{m}{k} d(T^k x, T^k y)^p \le \sum_{\substack{0 \le k \le m \\ k \text{ odd}}} (-1)^k \binom{m}{k} d(T^k x, T^k y)^p, \quad \forall\, x,y \in X.$$

Similarly a self mapping T on a metric space (X,d) is (m,p) contractive if and only if

$$\sum_{0 \le k \le m} (-1)^k \binom{m}{k} d(T^k x, T^k y)^p \ge 0$$

$$\Leftrightarrow \sum_{\substack{0 \le l \le m \\ k \text{ even}}} \binom{m}{k} d(T^k x, T^k y)^p \ge \sum_{\substack{0 \le k \le m \\ k \text{ odd}}} \binom{m}{k} d(T^k x, T^k y)^p, \quad \forall\, x,y \in X.$$

By taking the limit as $p \to \infty$ we get the following definition.

Definition 2.1. [20, Definition 3.1] Let $m \in \mathbb{N}$. An mapping T acting on a metric space X is called an

(1) (m,∞)-expansive if for all $x,y \in X$

$$\max_{\substack{0 \le k \le m \\ k \text{ even}}} d(T^k x, T^k y) \le \max_{\substack{0 \le k \le m \\ k \text{ odd}}} d(T^k x, T^k y)$$

(2) (m,∞)-hyperexpansive if T is (k,∞)-expansive for $k = 1, \cdots, m$.

(3) completely ∞-hyperexpansive if and only if T is (k,∞)-expansive for all $k \in \mathbb{N}$.

(4) (m,∞)-contractive if, and only if, for all $x,y \in X$

$$\max_{\substack{0 \le k \le m \\ k \text{ even}}} d(T^k x, T^k y) \ge \max_{\substack{0 \le k \le m \\ k \text{ odd}}} d(T^k x, T^k y).$$

(5) (m,∞)-hypercontractive if, and only if T is (k,∞)-contractive for $k = 1, \cdots, m$.

(6) completely ∞-hypercontractive if and only if T is (k,∞)-contractive for all $k \in \mathbb{N}$.

Remark 2.1. Observe that every (m,∞)-isometric mapping is an (m,∞)-expansive and an (m,∞)-contractive mapping.

Example 2.1. Let $X = [0,\infty)$ endowed with the standard metric $d(x,y) = |x-y|$ for all $x,y \in X$. Define the mapping $T : X \to X$ by $Tx = x^n + 3x + 7$. A simple calculation shows that

$$\begin{aligned} d(Tx,Ty) &= |Tx - Ty| = |x^n - y^n + 2(x-y| = |(x-y)\sum_{0 \le k \le n-1} x^{n-1-k} y^k + 2(x-y)| \\ &= |x-y||(\sum_{0 \le k \le n-1} x^{n-1-k} y^k + 2)| \\ &\ge 2|x-y| \\ &\ge d(x,y). \end{aligned}$$

Hence, T is an $(1,\infty)$-expansive mapping.

Example 2.2. Let $X = \mathbb{R}$ be equipped with the Euclidean metric $d(x,y) = |x-y|$ for all $x,y \in X$. Define $T : X \to X$ by $Tx = 2x$. A simple computation shows that T is $(2,\infty)$-*contractive* and $(3,\infty)$-*expansive*.

Proposition 2.1. [20, Proposition 3.1]. *Let $T : X \to X$ be a map,the following properties hold*

(1) *T is (m,∞)-expansive if and only if*

$$\max_{\substack{j \le k \le j+m \\ k \text{ even}}} d(T^k x, T^k y) \le \max_{\substack{j \le k \le j+m \\ k \text{ odd}}} d(T^k x, T^k y), \ \forall \ x,y \in X, \ \forall$$ [redacted] $\in \mathbb{N}_0$.

(2) T is (m,∞)-contractive if, and only if

$$\max_{\substack{j\le k\le j+m\\ k\ even}} d(T^kx,T^ky) \ge \max_{\substack{j\le k\le j+m\\ k\ odd}} d(T^kx,T^ky),\ \forall\ x,y\in X,\ \forall\ j\in\mathbb{N}_0.$$

Remark 2.2. (i) every $(2,\infty)$-expansive mapping is a $(1,\infty)$-expansive mapping.

(ii) Every (m,∞)-expansive mapping is injective.

(iii) An (m,∞)-expansive map is mot in general an $(m+1,\infty)$-expansive, as we shown in the following example.

Example 2.3. Consider the usual metric $d(x,y)=\mid x-y\mid$ on $X=\mathbb{R}$, and let $T:\mathbb{R}\to\mathbb{R}$ the map defined by $Tx=3x+6$. A simple calculation shows that $d(Tx,Ty)\ge d(x,y)$ and $d(Tx,Ty)\le \max\{d(T^2x,T^2y),d(x,y)\}$. So that, T is $(1,\infty)$-expansive which is not $(2,\infty)$-expansive.

Remark 2.3. (i) Every $(2,\infty)$-isometric mapping is an completely ∞-hyperexpansive.

(ii) Every $(m+1,\infty)$-hyperexpansive mapping is an (m,∞)-hyperexpansive mapping .

(ii) Every $(m+1,\infty)$-hypercontractive mapping is an (m,∞)-hypercontractive mapping

Proposition 2.2. *Let T be an mapping acting on a metric space X. Then T is an $(2,\infty)$-expansive mapping if and only if T is an $(2,\infty)$-isometric mapping.*

Proof. Assume that T is an $(2,\infty)$-expansive mapping. Then if follows that for all $x,y\in X$

$$d(Tx,Ty)\ge \max\{d(T^2x,T^2y),d(x,y)\}.$$

It holds

$$d(Tx,Ty)\ge d(T^2x,T^2y)\quad\text{and}\quad d(Tx,Ty)\ge d(x,y), forall\ x,y\in X.$$

This immediately yields

$$d(Tx,Ty)=d(T^2x,T^2y)\ge d(x,y)\ \ \forall\ x,y\in X.$$

Hence we conclude that T is an $(2,\infty)$-isometric mapping by [3, Proposition 2.4]. The converse is obvious. □

Corollary 2.1. *Every $(2,\infty)$-expansive mapping is an completely ∞-hyperexpansive.*

Proof. Let T be an $(2,\infty)$-expansive mapping. Then, we have T is a $(1,\infty)$-expansive and a $(2,\infty)$-isometry. Consequently, T is an (k,∞)-expansive mapping for all $k\in\mathbb{N}$. □

Corollary 2.2. *A power of an $(2,\infty)$-expansive mapping is again an $(2,\infty)$-expansive mapping.*

Proof. The proof is an immediate consequence of Proposition 2.2 and [3, Proposition 2.5]. □

Proposition 2.3. *Let $T : X \longrightarrow X$ be a mapping such that T^2 is an $(1,\infty)$-isometry. Then the following statement hold*

(i) T is an (m,∞)-expansive mapping if and only if T is an $(1.\infty)$-expansive mapping.

(ii) T is an (m,∞)-contractive mapping if and only if T is an $(1,\infty)$-contractive mapping.

Proof. (i) Assume that T is an (m,∞)-expansive. By the assumption that T^2 is an $(1,\infty)$-isometry, if follows that $d(T^2x,T^2y) = d(x,y)$ for all $x,y \in X$. This implies that

$$d(T^{2k}x,T^{2k}y) = d(x,y) \text{ and } d(T^{2k+1}x,T^{2k+1}y) = d(Tx,Ty) \;\; \forall\, x,y \in X,\; \forall\, k \in \mathbb{N}_0.$$

Since T is an (m,∞)-expansive, we have that for all $x,y \in X$

$$\max_{\substack{0\le k\le m \\ k \text{ even}}} d(T^kx,T^ky) \le \max_{\substack{0\le k\le m \\ k \text{ odd}}} d(T^kx,T^ky) \Longrightarrow d(x,y) \le d(Tx,Ty).$$

Consequently T is an $(1,\infty)$-expansive mapping.

Conversely, assume that T is an $(1,\infty)$-expansive mapping. Then $d(Tx,Ty) \ge d(x,y)$ for all $x,y \in X$ and therefore $d(x,y) = d(T^2x,T^2y) \ge d(Tx,Ty)$. We deduce that $d(Tx,Ty) = d(x,y)$ for all $x,y \in X$. Consequently, T is an $(1,\infty)$-isometry and so that T is an (m,∞) isometry (see Theorem 2.1 in [3]). Hence, T is an (m,∞)-expansive mapping.

(ii) This statement is proved in the same way as in the statement (i). □

Proposition 2.4. *Let $T : X \longrightarrow X$ be a mapping such that $T^2 = T$. Then the following statement hold*

(i) T is an (m,∞)-expansive mapping if and only if T is an $(1,\infty)$-expansive mapping.

(ii) T is an (m,∞)-contractive mapping if and only if T is an $(1,\infty)$-contractive mapping.

Proof. From that assumption that $T^2 = T$ it follows immediately that for all $x,y \in X$

$$\max_{\substack{0\le k\le m \\ k \text{ even}}} d(T^kx,T^ky) \le \max_{\substack{0\le k\le m \\ k \text{ odd}}} d(T^kx,T^ky) \Leftrightarrow d(x,y) \le d(Tx,Ty)$$

and

$$\max_{\substack{0\le k\le m \\ k \text{ even}}} d(T^kx,T^ky) \ge \max_{\substack{0\le k\le m \\ k \text{ odd}}} d(T^kx,T^ky) \Leftrightarrow d(x,y) \ge d(Tx,Ty).$$

Hence, the statements (i) and (ii) hold. □

In the following theorem, we generalize [20, Proposition 2.8] as follows.

Theorem 2.1. *Let T be an mapping on a metric space X such that T is bijective. The following statements hold.*

(i) If T is an (m,∞)-expansive, then T^{-1} is an (m,∞)-expansive for m even and an

(m,∞)-contractive for m odd.

(ii) If T is an (m,∞)-contractive, then T^{-1} is an (m,∞)-contractive for m even and an (m,∞)-expansive for m odd.

Proof. (i) Assume that T is a bijective an (m,∞)-expansive mapping. It follows that for all $x,y \in X$

$$\max_{\substack{0\le k\le m\\ k \text{ even}}} d\left(T^k x, T^k y\right) \le \max_{\substack{0\le k\le m\\ k \text{ odd}}} d\left(T^k x, T^k y\right). \tag{2.1}$$

Replacing x by $T^{-m}x$ and y by $T^{-m}y$ in (2.1), we get for all $x,y \in X$,

$$\max_{\substack{0\le k\le m\\ k \text{ even}}} d\left((T^{-1})^{m-k}x, (T^{-1})^{m-k}y\right) \le \max_{\substack{0\le k\le m\\ k \text{ odd}}} d\left((T^{-1})^{m-k}x, (T^{-1})^{m-k}y\right).$$

We obtain the following conclusions:

If m is even then, by equation (2.1) we have for all $x,y \in X$

$$\max_{\substack{0\le j\le m\\ j \text{ even}}} d\left((T^{-1})^j x, (T^{-1})^j y\right) \le \max_{\substack{0\le j\le m\\ j \text{ odd}}} d\left((T^{-1})^j x, (T^{-1})^j y\right)$$

and so that T^{-1} is an (m,∞)-expansive mapping.

If m is odd we have for all $x,y \in X$

$$\max_{\substack{0\le j\le m\\ j \text{ even}}} d\left((T^{-1})^j x, (T^{-1})^j y\right) \ge \max_{\substack{0\le j\le m\\ j \text{ odd}}} d\left((T^{-1})^j x, (T^{-1})^j y\right)$$

and so that T^{-1} is an (m,∞)-contractive mapping.

(ii) This statement is proved in the same way as in the statement (i). □

Corollary 2.3. *Let $T : X \to X$ be an bijective mapping. The following statements hold.*

(i) If T is an $(2,\infty)$-expansive mapping, then T is an $(1,\infty)$-isometry.

(ii) If T is an $(2,\infty)$-contractive mapping, then T is an $(1,\infty)$-isometry.

Proof. Assume the T is an $(2,\infty)$-expansive mapping. Then it follows that $d\left(Tx,Ty\right) \ge d\left(x,y\right)$ for all $x,y \in X$. On the other hand, by the fact that T^{-1} is bijective $(2,\infty)$-expansive, we have by Theorem 2.1 that T^{-1} is a $(2,\infty)$-expansive and hence $d\left(T^{-1}x, T^{-1}y\right) \ge d\left(x,y\right)$ for all $x,y \in X$. This means that $d\left(x,y\right) \ge d\left(Tx,Ty\right)$ for all $x,y \in X$. Consequently, $d\left(Tx,Ty\right) = d\left(x,y\right)$ for all $x,y \in X$, which shows that T is an $(1,\infty)$-isometry as required.

(ii) This statement is proved in the same way as in the statement (i). □

Theorem 2.2. *For $i = 1,2,...,n$, let (X_i, d_i) be a metric space and let $T_i : X_i \to X_i$ be a map, $m_i \geq 1$. Denote by $X = X_i \times X_2 \times ... \times X_n$ the product space endowed with the product distance $d\big((x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n)\big) := \max_{1\leq i\leq n} \Big(d_i\big(x_i, y_i\big) \Big)$. Let $T := T_1 \times T_2 \times ... \times T_n : X \to X$ be a mapping defined by*

$$T(x_1, \cdots, x_n) := (T_1x_1, T_2x_2, \cdots, T_nx_n).$$

The following statements hold.

(i) If each T_i is an (m_i, ∞)-hyperexpansive for $i = 1,2,\cdots,n$, then T is an (m, ∞)-expansive, where $m = \min(m_1, \cdots, m_n)$.

(ii) If each T_i is an (m_i, ∞)-hypercontractive for $i = 1,2,\cdots,n$, then T is an (m, p)-contractive, where $m = \min(m_1, \cdots, m_n)$.

(iii) If each T_i is an completely ∞-hyperexpansive for $i = 1,2,\cdots,n$, then so that T.

(iv) If each T_i is completely ∞-hypercontractive for $i = 1,2,\cdots,n$, then so that T.

Proof. (i) Let $m = \min(m_1, m_2, \cdots, m_n)$ and consider for all $x, y \in X$

$$\begin{aligned}
\max_{\substack{0\leq k\leq m \\ k \text{ even}}} d\big((T^kx, (T^ky)\big) &= \max_{\substack{0\leq k\leq m \\ k \text{ even}}} \Big(\max_{1\leq i\leq n} \big\{d_i\big((T_i^kx_i, (T_i^ky_i)\big\}\Big) \\
&= \max_{1\leq i\leq n} \Big(\max_{\substack{0\leq k\leq m \\ k \text{ even}}} \big\{d_i\big((T_i^kx_i, (T_i^ky_i)\big\}\Big)
\end{aligned}$$

Since T_i is an (m_i, ∞)-hyperexpansive for $i = 1,2,...,n$, it follows that T_i is an (m, ∞)-expansive for $i = 1,2,\cdots,n$ and hence

$$\begin{aligned}
\max_{\substack{0\leq k\leq m \\ k \text{ even}}} d\big(T^kx, T^ky\big) &\leq \max_{1\leq i\leq n} \Big(\max_{\substack{0\leq k\leq m \\ k \text{ odd}}} \big\{d_i\big(T_i^kx_i, T_i^ky_i\big)\big\}\Big) \\
&= \max_{\substack{0\leq k\leq m \\ k \text{ odd}}} \Big(\max_{1\leq i\leq n} \big\{d_i\big(T_i^kx_i, T_i^ky_i\big)\big\}\Big).
\end{aligned}$$

Thus, we have

$$\max_{\substack{0\leq k\leq m \\ k \text{ even}}} d\big(T^kx, T^ky\big) \leq \max_{\substack{0\leq k\leq m \\ k \text{ odd}}} d\big(T^kx, T^ky\big).$$

Consequently, T is an (m, ∞)-expansive mapping.

(ii) This statement follows from the statement in (i) by reversing the inequality above.

(iii) Suppose that each T_i is an completely ∞-hyperexpansive for each $i = 1,2,...,n$, and hence each T_i is an (k,∞)-expansive for any $k \in \mathbb{N}$. As a consequence of this observation , one can deduce the following inequality for all $x, y \in X$

$$\begin{aligned}
\max_{\substack{0 \le j \le k \\ j \text{ even}}} d(T^j x, T^j y) &= \max_{\substack{0 \le j \le k \\ j \text{ even}}} \left(\max_{1 \le i \le n} d_i(T_i^j x_i, T_i^j y_j) \right) \\
&= \max_{1 \le i \le n} \left(\max_{\substack{0 \le j \le k \\ j \text{ even}}} d_i(T_i^j x_i, T_i^j y_j) \right) \\
&\le \max_{\substack{0 \le j \le k \\ j \text{ odd}}} d(T^j x, T^j y) \quad \forall\ k \in \mathbb{N}.
\end{aligned}$$

From which the statement in (iii) follows.

(iv) This statement is proved in the same way as in the statement (iii). □

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